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$$\therefore\ dr/r=\frac{ndv}{(1+v^2)\{(1+v^2)^{\frac{1}{2}}-nv\}}.$$

$$\therefore\ \log r=\log\left\{\frac{(1+v^2)^{\frac{1}{2}}}{(1+v^2)^{\frac{1}{2}}-nv}\right\}+\log A.$$

$$\therefore\ r=\frac{A(1+v^2)^{\frac{1}{2}}}{(1+v^2)^{\frac{1}{2}}-nv},\ =\frac{A(r^2+p^2)^{\frac{1}{2}}}{(r^2+p^2)^{\frac{1}{2}}-nr}.$$

$$\therefore\ r\left\{r^2+\left(\frac{dr}{d\theta}\right)^2\right\}^{\frac{1}{2}}-nr^2=A\left\{r^2+\left(\frac{dr}{d\theta}\right)^2\right\}^{\frac{1}{2}}$$

$$\therefore\ d\theta=\pm\frac{(r-A)dr}{r\sqrt{n^2r^2-(r-A)^2}},\ =\pm\frac{(r-A)dr}{nr^2\sqrt{1-\{(r-A)/(nr)\}^2}}.$$

$$\therefore\ \theta=\pm\frac{1}{\sqrt{n^2-1}}\log[(n^2-1)r+A+\sqrt{(n^2-1)\{r^2n^2-(r-A)^2\}}\,]$$

$$\mp\sin^{-1}\left(\frac{r-A}{nr}\right)+B.$$

$$\therefore\ \tan^{-1}(y/x)\pm\sin^{-1}\left(\frac{\sqrt{x^2+y^2}-A}{n\sqrt{x^2+y^2}}\right)-B,=\pm\frac{1}{\sqrt{n^2-1}}\log[(n^2-1)\sqrt{x^2+y^2}$$

$$+A+\sqrt{(n^2-1)\{n^2(x^2+y^2)-(\sqrt{x^2+y^2}-A)^2\}}\,],$$

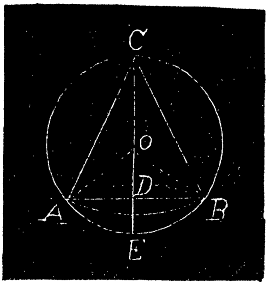
where A and B are constants of integration.

60. Proposed by SETH PRATT, C. E., Assyria, Mich.

To remove $(1/a)$th of the volume of a sphere of a given radius by a conical hole, whose axis is the axis of the sphere, and whose vertex is at the surface of the sphere. Required the height of the cone and the diameter of its base.

I. Solution by J. SCHEFFER, A. M., Hagerstown, Md.

Let O be the center of the sphere, ABC the cone, $AO=r$, $DE=x$, $AD=y$; then the volume of spherical segment



$$ABE=\frac{\pi x^2}{3}(3r-x),$$

and that of the cone

$$ABC=\frac{\pi y^2(2r-x)}{3};$$

by condition, therefore,

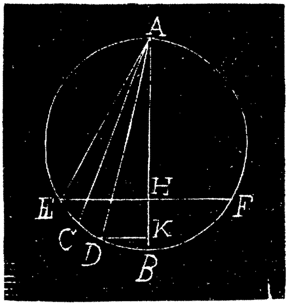
$\frac{\pi x^2}{3}(3r-x)+\frac{\pi y^2}{3}(2r-x)=(1/a).\frac{4}{3}\pi r^3$. Substituting $y^2=2rx-x^2$, we obtain the final equation, $x^2-4rx=-(4r^2/a)$, whence $x=2r\{1-[1-(1/a)]^{\frac{1}{2}}\}$.

$\therefore$ Height of cone $2r-x=2r[1-(1/a)]^{\frac{1}{2}}$, and $y=2r\{[1-(1/a)]^{\frac{1}{2}}-[1-(1/a)]\}^{\frac{1}{2}}$.

II. Solution by C. W. M. BLACK, A. M., Professor of Mathematics in Wesleyan Academy, Wilbraham, Mass.

Figure shows section of sphere through axis, with $AEBF$ as section of hole.

Let $h=AH=$height of cone ; $b=FH=$radius of base ; $dA=ACD=$elements of area AEB ; $x(=DK)$ is perpendicular to AB ; $\theta=\angle BAD$.



Then $dA=\frac{1}{2}\overline{AD}^2d\theta=2r^2\cos^2\theta d\theta$.

Center of gravity of ADC is at distance $2x/3$ from AB. The element of volume found by revolving ADC about AB, or $dV=2\pi\times(2x/3)\times 2r^2\cos^2\theta d\theta$.

But $x=AD\sin\theta=2r\cos\theta\sin\theta$.

$\therefore$ $dV=(16\pi/3)r^3\cos^3\theta\sin\theta d\theta$;

$$\cos\angle EAH=\frac{AH}{AE}=\frac{h}{\sqrt{2rh}}=\sqrt{\frac{h}{2r}}.$$

$$\therefore \text{ Volume } AEBF=\frac{16\pi r^3}{3}\int_0^{\cos^{-1}\sqrt{(h/2r)}}\cos^3\theta\sin\theta d\theta,=(\pi r/3)(4r^2-h^2),$$

which equals $(1/a)$th of volume of sphere, or $4\pi r^3/3a$.

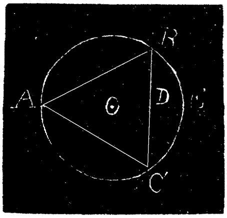
$\therefore$ $h=2r\sqrt{[1-(1/a)]}$.

$$\text{Diameter}=2b=2\sqrt{h(2r-h)}=4r\sqrt{\sqrt{1-(1/a)}[1-\sqrt{1-(1/a)}]}.$$

Volume $AEBF$ can be as easily found by geometry without the use of calculus.

III. Solution by G. B. M. ZERR, A. M., Ph. D., President and Professor of Mathematics, Russell College, Lebanon, Va.

Let $AO=r$, $DO=y$, $DB=x$. Volume of cone $ABC=\frac{1}{3}\pi(r+y)x^2$; volume of segment $BDCE=\frac{1}{3}\pi(r-y)^2(2r+y)$.



$\therefore$ $\frac{1}{3}\pi(r+y)x^2+\frac{1}{3}\pi(r-y)^2(2r+y)=(4\pi r^3/3a)$, but $x^2=r^2-y^2$.

$\therefore$ $(r+y)(r^2-y^2)+(r-y)^2(2r+y)=(4r^3/a)$.

$\therefore$ $y^2+2ry=(3ar^2-4r^2)/a$.

$$\therefore y=\pm 2r\sqrt{\frac{a-1}{a}}-r.$$

The plus sign alone is admissible. $\therefore y=2r\sqrt{\frac{a-1}{a}}-r$.

$\therefore$ Altitude $=r+y$, $=2r\sqrt{\frac{a-1}{a}}$; $2x$=diameter of base$=2\sqrt{r^2-y^2}$.

$$2x=2r\sqrt{\sqrt{\frac{a-1}{a}}-\frac{a-1}{a}},\ =2r\sqrt[4]{\frac{a-1}{a}}\sqrt{\frac{\sqrt{a}-\sqrt{a-1}}{\sqrt{a}}}.$$

MECHANICS.

Conducted by B. F. FINKEL, Springfield, Mo. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

50. Proposed by J. SCHEFFER, A. M., Hagerstown, Md.

A plane quadrilateral $ABCD$ in the vertical wall of a cistern, filled with water, has its four vertices A, B, C, D at the distances 10 feet, 4 feet, 5 feet, and 7 feet respectively, from the surface of the water. The projections of AB, BC, CD upon the surface are respectively 2 feet, 3 feet, and 1 foot. Find the pressure of the water upon the quadrilateral, and the position of the center of mean pressure.

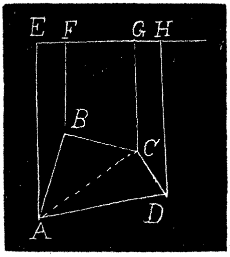
Solution by J. C. NAGLE, A. M., M. C. E., Professor of Civil Engineering in the State Agricultural and Mechanical College, College Station, Texas, and the PROPOSER.

In the figure, $AE=10$, $BF=4$, $CG=5$, $DH=7$, $EF=2$, $FG=3$, $GH=1$. The coördinates of the vertices A, B, C, and D with respect to EH and EA as axes are respectively, (10, 0), (2, 4), (5, 5), and (6, 7).

Hence, the area of $ABCD$

$$=\tfrac{1}{2}\left[\begin{vmatrix} x_1 & x_2 \\ y_1 & y_2 \end{vmatrix}+\begin{vmatrix} x_2 & x_3 \\ y_2 & y_3 \end{vmatrix}+\begin{vmatrix} x_3 & x_4 \\ y_3 & y_4 \end{vmatrix}\right]=17\tfrac{1}{2};$$

area of triangle $ABC=10$; and area of $ACD=7\frac{1}{2}$.



Distance of center of gravity of $\triangle ABC$ from $EH=\frac{1}{3}(10+4+5)=6\frac{1}{3}$, and distance of center of gravity of $\triangle ACD$ from $EH=\frac{1}{3}(10+4+7)=7\frac{1}{3}$. Denoting the distance of center of gravity of $ABCD$ by z, we have $17\frac{1}{2}z=10\times6\frac{1}{3}+7\frac{1}{2}\times7\frac{1}{3}$. Hence, $z=6\frac{16}{21}$. Hence, pressure of water upon $ABCD=17\frac{1}{2}\times6\frac{16}{21}w=\frac{355}{2}w$, w denoting the weight of a cubic foot of water.

For $u=62\frac{1}{2}$ pounds, we find the pressure to be $11093\frac{3}{4}$ pounds.

Let AD represent the surface of the water, $ABCD$ a rectangle, BCE a right triangle, $AE=a$, $CD=b$, $AD=c$. Then, omitting w, the moment of the